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THE EFFECTIVE PERMEABILITY TENSOR OF HEAVILY INHOMOGENEOUS GROUNDS

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A method is developed for the construction of the effective permeability tensor in an anisotropic model of nondeformable grounds with a complicated structure, consisting of systems of mutually parallel layers, joints, and slightly permeable screens (interlayers), enclosed in one another (each layer of one system consists of arbitrary oriented layers, joints and screens of another). A new filtration model of joints and screens is suggested.

Filtration processes are usually associated with expressed inhomogeneity of natural grounds because of their stratified structure, jointing, presence of slightly pervious screens, etc. [1, 2]. Tectonic joints and screens often appear at the interface of heterogeneous layers and form substantially regular systems [2, 3] such as free and sealed slits, veins, and interlayers.

Accurately posed filtration problems in stratified grounds (conjugation problems) were solved in a general form for two, three, and four homogeneous zones (layers), separated by straight lines and circumferences (see the review in [4]). Numerous approximate methods have been developed for the conjugation problems with an arbitrary number of layers ([5-7] et al.). For systems of layers enclosed in one another the conjugation problem is essentially complicated, and in its solution the properties of grounds are globally averaged, i.e. their effective filtration parameters are determined. In [8-10] tensors of effective permeability were found for one system and two systems of mutually orthogonal periodical of layers. For jointly (cracked) porous media a model was suggested in [11] as two mutually penetrating continua with an average scalar permeability of blocks and joints. The effective permeability tensors were constructed for nondeformable [2, 3] and deformable [12, 13] jointly grounds. The authors quoted considered relatively simple structures of jointly grounds when the blocks are either impermeable or permeable and homogeneous. Moreover, the liquid flow in a joint is based on the model of viscous liquid motion through a channel with impermeable walls, and according to the model, hydraulic permeability of the joint is proportional to the cube of its opening (Boussinesq's formula) [2, 3, 6, 9, 12], that is permeability of fine joints is negligibly small. This model, based on the lubrication theory [14], is sufficiently idealized in the filtration theory where the wall surfaces of the joints are usually permeable and can be in contact at several points, the joints being partially or fully filled with debris materials [2]. As regards filtration, a joint is a layer, whose thickness is much smaller and permeability much larger than the characteristic parameters of the ground.

The article describes a method for constructing the effective permeability tensors for nondeformable grounds, which consist of arbitrary oriented anisotropic inhomogeneous systems of layers, joints and screens enclosed into one another with an arbitrary depth of enclosure. Moreover, a filtration model of joints and screens is suggested in the form of degenerating layers of

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infinitely small thickness and infinitely large (or small) permeability. In particular cases the expressions found give the known results based on different considerations.

Directional Permeability. In the space $x_1x_2x_3$ consider anisotropic ground, in which the components v_i of the filtration velocity satisfy the general linear system

$$v_i = \sum_{j=1}^{3} K_{ij} \frac{\partial \varphi}{\partial x_j}, \quad i = \overline{1, 3}, \tag{1}$$

where

$$K_{ij} = K_{ji}. \tag{2}$$

Inversion of Eq. (1) gives the relation between the filtration velocity modulus v having a fixed direction $s = (\alpha_i)$ and the respective derivative $\partial\varphi/\partial s$ as

$$v = K_s \frac{\partial \varphi}{\partial s}, \tag{3}$$

where

$$K_s = A\Big(\sum_{i,j=1}^{3} A_{ij}\alpha_i\alpha_j\Big)^{-1}. \tag{4}$$

Hence, along the streamline the Darcy law (3) is satisfied where K_s is the directional permeability. From the necessary inequality $K_s > 0$ we find with Silvester's criterion [15] the inequalities

$$K_{ii} > 0, \quad A_{ii} > 0, \quad A > 0. \tag{5}$$

Modeling of Laminated Anisotropic Grounds. In the space xyz consider the ground, consisting of m anisotropic layers D_n $(z_{n-1} < z < z_n)$ with the permeability tensors $T_n = (K_{ij}^n)$, $n = 1, \ldots, m$, where K_{ij}^n are arbitrary constants, satisfying the conditions (2), (5), and the system of layers may regularly occur along the z-axis. In the region D_0 $(z < z_0)$ with permeability K_0 consider an arbitrary translational flow, prescribed by the potential $\varphi_0 = ax + by + cz/K_0$. While solving the corresponding conjugation problem

$$\operatorname{div}(T_n \nabla \varphi_n) = 0, \quad z = z_n\colon \ [\varphi_n] = 0, \ [v_3^n] = 0, \tag{6}$$

write the potentials φ_n in layers D_n as $\varphi_n = ax + by + c_n z + d_n$ where

$$c_n = \frac{1}{K_{33}^n}(c - aK_{13}^n - bK_{23}^n), \quad d_n = \sum_{\nu=1}^{n} c_\nu l_\nu - c_n z_n + \frac{1}{K_0} c z_0,$$

$l_n = z_n - z_{n-1}$. Hence, with (1) in view, find the components v_i^n of the filtration velocity v_n, its constituting cosines $\alpha_i^n = v_i^n/|v_n|$, the lengths of the sections $s_n = l_n/\alpha_3^n$ of the broken streamline, their projections $r_i^n = \alpha_i^n s_n$, $i = 1, 2$, onto the axes x and y in the layers D_n and the time, $t = \Sigma_n s_n/|v_n|$, during which a liquid particle passes the whole streamline (here and in the following $n = 1, \ldots, m$ in the sums).

In the modeling suggested a homogeneous inclusion is substituted for the stratified inclusion D so that at the inclusion boundaries $z = z_0$, $z = z_m$ the main parameters of the flow (potential values, a flow through the boundary, inlet and outlet points for liquid particles, and time of their passing the region D) remain unchanged. From the boundary values $\varphi = \varphi_0$ at $z = z_0$ and $\varphi = \varphi_0$ at $z = z_m$ find the average potential, satisfying any of Eqs. (6) with constant coefficients

$$\varphi = ax + by + \frac{z}{l}(c\sigma - aN_1 - bN_2) + \Big(\frac{c}{K_0} - \frac{1}{l}\sum_n c_n l_n\Big) z_0,$$

where

$$N_i = \sum_n \frac{l_n K_{i3}^n}{K_{33}^n}, \quad l = \sum_n l_n, \quad \sigma = \sum_n \frac{l_n}{K_{33}^n}. \tag{7}$$

Here a chord of the length s: $s^2 = \Sigma_{i=1}^{2} (\Sigma_n r_i^n)^2$ with the constituent cosines $\alpha_i = (1/s)\Sigma_n r_i^n$ is substituted for the broken streamline. Calculating the mean velocity v = s/t and the derivative $\partial\varphi/\partial s$, where s = ($\alpha_i$) from the Darcy law (3), we find the average permeability of the homogeneous model in the direction s as Eq.(4) where

$$K_{11} = \frac{1}{l}\left(R_{22} + \frac{N_1^2}{\sigma}\right); \; K_{22} = \frac{1}{l}\left(R_{11} + \frac{N_2^2}{\sigma}\right);$$
$$K_{33} = \frac{l}{\sigma}; \; K_{12} = \frac{1}{l}\left(\frac{N_1 N_2}{\sigma} - R_{12}\right); \; K_{i3} = \frac{N_i}{\sigma}. \tag{8}$$

Here

$$R_{ij} = \sum_n \frac{l_n A_{ij}^n}{K_{33}^n}, \tag{9}$$

and K_{ij} is consistent with the conditions (5), i.e. a laminated anisotropic inclusion can be modeled by homogeneous ground with components of the effective permeability tensor as in Eq. (8).

Modeling of Stratified Anisotropic Inhomogeneous Grounds with Joints and Screens. Let the layers D_n alternate with joints and screens along the planes z = const. Let joints and screens be substituted by homogeneous layers with thickness b_ν, h_μ and permeabilities q_ν, P_μ, respectively, $\nu = 1, \ldots, m_1$, $\mu = 1, \ldots, m_2$. The resultant stratified ground will be modeled by Eqs. (8), followed by passing to the limit at b_ν, h_μ, $P_\mu \to 0$, $q_\nu \to \infty$. It is assumed here that

$$A_\nu = \lim b_\nu q_\nu, \; B_\mu = \lim \frac{h_\mu}{P_\mu}, \tag{10}$$

where A_ν, B_ν are finite parameters of joints and screens, characterizing their opening and permeability. In practical calculations, with estimates b_ν, h_μ, $P_\mu \ll 1$, $q_\nu \gg 1$, taken into consideration, $A_\nu = b_\nu q_\nu$ and $B_\mu = h_\mu/P_\mu$ can be assumed. Hence, the components of the effective permeability tensors for the grounds considered will be found from Eqs. (8) where N_i, l, and R_{12} have the form of Eqs. (7), (9),

$$\sigma = \sum_n \frac{l_n}{K_{33}^n} + \sum_\mu B_\mu, \; R_{ii} = \sum_n \frac{l_n A_{ii}^n}{K_{33}^n} + \sum_\nu A_\nu. \tag{11}$$

If there are no joints and screens ($A_\nu = B_\mu = 0$) and layers D_n are isotropic, the known expressions, obtained from different considerations [8], follow from Eqs. (8).

Consider a stratified anisotropic inhomogeneous inclusion $D(\alpha < z < \beta)$ with the permeability tensor ($f_{ij}(z)$) in the presence of joints and screens z = const, where $f_{ij}(z)$ are piecewise continuous functions. As before, dividing the region D into layers $D_n(z_{n-1} < z < z_n)$ and passing in the integral sums (7), (9), and (11) to the limit at $l_n = \Delta z_n \to 0$, find the components of the effective permeability tensor of the inclusion D from Eqs. (8) where

$$R_{ii} = \int_\alpha^\beta \frac{A_{ii}}{f_{33}} dz + \sum_\nu A_\nu; \; \sigma = \int_\alpha^\beta \frac{dz}{f_{33}} + \sum_\mu B_\mu;$$
$$R_{12} = \int_\alpha^\beta \frac{A_{12}}{f_{33}} dz; \; N_i = \int_\alpha^\beta \frac{f_{i3}}{f_{33}} dz; \; l = \beta - \alpha;$$

and A_{ij} are cofactors of the matrix ($f_{ij}(z)$).

Method of Successive Modeling. Anisotropic models of grounds consisting of anisotropic layers with joints and screens can be constructed from Eqs. (7)-(11). In the model, anisotropic ground in each layer may also be considered as a model of a stratified ground with joints and screens, etc. Thus, successive application of Eqs. (7)-(11) gives models of stratified ground systems enclosed in one another (first, models are constructed for the innermost layers with joints and screens and in subsequent modeling the number of enclosed systems decreases at each step).

Consider a particular case. In homogeneous ground with permeability K, let there be N systems of joints, which in each system are parallel to one another, spaced by a distance l_n and having parameters A_n and vectors of the normal $\boldsymbol{\nu}_n = (\alpha_{n1}, \alpha_{n2}, \alpha_{n3})$ to the joint planes, n = 1, ..., N. Successive modeling gives the final components of the effective permeability tensor of the ground

$$K_{ii} = K + \sum_{n=1}^{N} L_n (1 - \alpha_{ni}^2),\ K_{ij} = -\sum_{n=1}^{N} L_n \alpha_{ni} \alpha_{nj},\ i \neq j, \tag{12}$$

where $L_n = A_n/l_n$. Hence, for impermeable blocks $K = 0$, according to the Boussinesq law we have $q_n = b_n^2/12$, Eqs. (10) and (12) coincide with the similar equations from [3].

Note that the effective permeability components (8) may be easily constructed with known parameters of layers, joints and screens. In the model the known isotropizing substitution reduces the equation of motion to the Laplace equation [3], because of which filtration can be investigated in new classes of grounds with a complicated structure.

NOTATION

$x_1x_2x_3$ and xyz, coordinates; v and v_i, modulus and components of filtration velocity; φ, potential; K_{ij}, permeability tensor components; s and α_1, unit vector and its coordinates; A, A_{ij}, and A_{ij}^n, determinant and cofactors of matrices (K_{ij}) and (K_{ij}^n); K_{ij}^n, components of the permeability tensor T_n of the layer D; K_0, penetrability factor of the region D_0; φ_n, potentials in the layers D_n; a, b, c, c_n, and d_n, flow parameters; $[\varphi]$, function discontinuity; v_i^n and α_i^n, components and constituting cosines of filtration velocity v_n; l_n, thickness of the layer D_n; s_n and r_i^n, lengths and projections of the sections of the broken streamline; s, length of the averaged rectilinear streamline; t, time; D, laminated inclusion with thickness l; R_{ij}, N_i, σ, parameters; z_{n-1} and z_n, boundary values of z in D_n; b_ν and h_μ, opening; q_ν and P_μ, permeability, A_ν and B_μ, parameters of joints and screens, respectively; $f_{ij}(z)$, components of effective permeability tensor; α and β, ends of the section of determining the functions f_{ij}; α_{ni}, coordinates of the vector of normal ν_n to the planes of joints; L_n, geometric parameters of joints; K, permeability of blocks; m, the number of layers; N, the number of joint systems.

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